Pairing correlations in statistical level densities within the micro-macroscopic approach

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In this report we present results for the statistical level density $\rho(E, N, Z)$ for several magic nuclei as function of the total energy *E*, and number of neutrons *N* and protons *Z* within the micro-macroscopic approach (MMA) [1], with main focus on pairing correlations. This level density ρ was improved at low excitation energy *U* [1]. The density ρ was derived as function of the excitation energy *U*, $\rho \propto S^{-\nu}I_{\nu}(S)$, in terms of the system entropy, $S = 2(aU)^{1/2}$, where *a* is the level density parameter, and $I_{\nu}(S)$ is the modified Bessel function of order ν . The orders $\nu = 2$ and $\nu = 3$ correspond to the cases of neglecting (MMA1) and dominating (MMA2) shell contributions, respectively. Taking into account the particle number fluctuations beyond the Bardeen-Cooper-Schrieffer (BCS) theory, the pairing gap Δ_0 can be considered as a smooth function of the particle number *A*. For the condensation energy E_c and the critical excitation energy U_c for a superfluid-normal phase transition, one can, respectively, use the well-known approximations, $E_c = 3a\Delta^2/(2\pi^2)$ and $U_c = aT_c^2 + \Delta^2/(4G)$, where $T_c = e^C \Delta_0/\pi$, with the Euler constant *C*, and *G* is the mean matrix element of residue interaction.



FIG. 1. Level density (in logarithms) as a function of excitation energy U for low energy states in the magic (close-shell) ⁴⁰Ca (a) and ⁵⁶Ni (b), semi-magic ⁵⁴Fe (c), and non-magic (open-shell) ⁵²Fe (d) nuclei. Solid lines show the results of the MMA approach for minimal values of LMS errors σ with pairing condensation being neglected. Dashed lines are the same but taking into account the pairing effect through the found condensation energy E_c . Blue dotted lines present the results of the Fermi gas approach. Experimental close circles are obtained from the ENSDF excitation energy data.

Fig. 1 presents a comparison between the results of the MMA approaches for relatively small excitation energies U, below neutron resonances, in four nuclei, ⁴⁰Ca (a), ⁵⁶Ni (b), ⁵⁴Fe (c), and ⁵²Fe (d), and the experimental data obtained from the database http://www.nndc.bnl.gov/ensdf, ENSDF. Close points with errors are obtained by using the energies and spins of excited states (with spin degeneracies) by the macroscopic sample method [1]. The results for MMA2a level density approach (with dominating contributions of shell and pairing corrections from [2]) in magic nucleus ⁴⁰Ca ($E_c = 2.3$ MeV, $U_c =$ 7.1 MeV) with the least mean square fit (LMSF) error $\sigma = 1.3$ agrees well with the experimental data obtained by least mean square (LMS) fitting using one physical parameter – the inverse level density parameter K = A/a. Those for the MMA2b approach (also with dominating contributions of these corrections but due to their large derivatives of the shell corrections over the chemical potential) in magic nucleus ⁵⁶Ni ($E_c = 0.8$ MeV, $U_c = 2.5$ MeV, $\sigma = 2.2$) are less in agreement with the experimental data when using similar LMS fitting. Pairing effects are larger for ⁴⁰Ca (a), see the difference between dashed and solid lines, in contrast to the ⁵⁶Ni (b) case. Condensation energies E_c and superfluid-normal phase transition energies U_c are marked by black and red arrows, respectively. The range between arrows for, ⁴⁰Ca, overlaps whole excitation energies while for the nickel, ⁵⁶Ni, there is no such an overlap. Therefore, we may predict that the pairing effects are easier to detect in ⁴⁰Ca than in ⁵⁶Ni. In contrast to these closeshell results, one has an intermediate situation for semi-magic ⁵⁴Fe (c) and open-shell ⁵²Fe (d) nuclei.

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